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View Factor Between Inclined Rectangles

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Introduction

SURFACE-to-surface radiant interchange analysis for surfaces separated by radiatively nonparticipating medium plays an important role in spacecraft thermal control system design, optical engineering, electronic package thermal design, solar energy applications, etc. The evaluation of view factors is necessary when the script-F, the absorption-factor, or the exchange-factor methods are used for the radiation analysis. A number of books on radiation heat transfer give formulas, tables, and charts for view factors for various geometric configurations.[1–4] Howell[5] presented tabulations of view factors for several geometric configurations. A configuration that is often encountered in practice is the arrangement of two arbitrarily positioned and sized rectangles with parallel boundaries where the planes containing the rectangles incline at an angle. The view factor algebra method could be used in this situation. Alternatively, Gross et al.[6] proposed a method that avoids the view factor algebra. The objective of this Note is to compare the computational performance of the two methods.

Analysis

The geometric arrangement of the surfaces is shown in Fig. 1. The view factor $F_{1\text{-}2}$ could be evaluated by the view factor algebra method by using the superposition of different view factors obtained for the basic configuration of two rectangles of equal y length with one common edge and included angle α. The view factor expression for this basic configuration is given by Sparrow and Cess,[1] and contains a number of terms including an integral that is evaluated numerically. The view factor algebra to find $F_{1\text{-}2}$ between two rectangles arbitrarily placed in perpendicular planes is given by Modest.[4] The same logic holds for inclined planes. This requires 16 evaluations of individual view factors to obtain $F_{1\text{-}2}$ in the general case. However, careful programming reduces the number of evaluations of view factors in certain cases of the geometrical arrangement.

The method suggested by Gross et al.[6] is the following:

$$A_1F_{1\text{-}2} = \sum_{l=1}^{2}\sum_{k=1}^{2}\sum_{j=1}^{2}\sum_{i=1}^{2}\left[(-1)^{(i+j+k+l)}\cdot G(x_i, y_j, \eta_k, \xi_l)\right] \tag{1}$$

$$\begin{aligned} G = {} & -\frac{\sin^2\alpha(\eta - y)}{2\pi}\int_\xi\left(\frac{\cos\alpha(x - \xi\cos\alpha) - \xi\sin^2\alpha}{\sin^2\alpha\sqrt{x^2 - 2x\xi\cos\alpha + \xi^2}}\right. \\ & \times\tan^{-1}\left(\frac{\eta - y}{\sqrt{x^2 - 2x\xi\cos\alpha + \xi^2}}\right) + \frac{\cos\alpha}{\sin^2\alpha(\eta - y)} \\ & \times\left\{\sqrt{\xi^2\sin^2\alpha + (\eta - y)^2}\tan^{-1}\left[\frac{x - \xi\cos\alpha}{\sqrt{\xi^2\sin^2\alpha + (\eta - y)^2}}\right]\right. \\ & \left. - \xi\sin\alpha\tan^{-1}\left(\frac{x - \xi\cos\alpha}{\sin\alpha}\right)\right\} + \frac{\xi}{2(\eta - y)} \\ & \left.\times\ \ell n\left[\frac{x^2 - 2x\xi + \xi^2 + (\eta - y)^2}{x^2 - 2x\xi + \xi^2}\right]\right)\mathrm{d}\xi \end{aligned} \tag{2}$$

Rectangle 1 is defined in x-y coordinates and rectangle 2 in η-ξ coordinates (see Fig. 1).

The function G is evaluated 16 times in Eq. (1). Note that the integral in Eq. (2) is obtained numerically because an analytical integration seems to be impossible. If numerical integration is used, the integral is evaluated between the limits ξ_1 and ξ_2 and the l loop is not needed. This calls for only eight evaluations of G. For α = 90 deg, analytical integration is possible, leading to a closed-form solution for $F_{1\text{-}2}$. Gross et

Table 1 Comparison of numerical results, case 1

	View factor algebra (20-panel Simpson) $nfun$ = 336		Gross et al.[6] (20-panel Simpson) $nfun$ = 168	
α, deg	$F_{1\text{-}2}$	CPU[a]	$F_{1\text{-}2}$	CPU[a]
10	0.002888	35.8	0.002888	35.0
30	0.012919	35.7	0.012919	34.7
45	0.014966	35.5	0.014966	34.4
60	0.013449	35.5	0.013449	33.9
90	0.007760	35.3	0.007760	34.4

[a]Time in milliseconds.

Table 2 Comparison of numerical results, case 2

	View factor algebra (20-panel Simpson) $nfun$ = 42		Gross et al.[6] (20-panel Simpson) $nfun$ = 168	
α, deg	$F_{1\text{-}2}$	CPU[a]	$F_{1\text{-}2}$	CPU[a]
10	0.055843	4.7	0.055833	29.5
30	0.081072	4.7	0.081072	29.2
45	0.072791	4.7	0.072791	28.8
60	0.059066	4.6	0.059066	28.4
90	0.032809	4.6	0.032809	28.0

[a]Time in milliseconds.

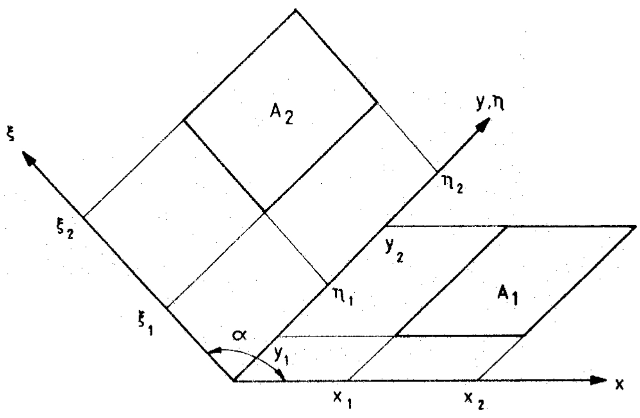

Fig. 1 Two rectangles in planes inclined at angle α.

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Table 3 Comparison of numerical results case 3[a]

α, deg	View factor algebra (20-panel Simpson) $nfun = 21$ $F_{1\text{-}2}$	CPU[b]	Gross et al.[6] (Q1DA) $F_{1\text{-}2}$	*nfun*	CPU[b]	Howell[5] (C-15, pp. 99–103) $F_{1\text{-}2}$
10	0.849606	2.4	0.849606	420	92.4	——
30	0.619028	2.4	0.619028	240	55.9	0.619028
45	0.483348	2.4	0.483348	240	55.5	0.483347
60	0.370905	2.4	0.370905	240	55.3	0.370905
90	0.200044	2.4	0.200044	240	54.1	0.200044

[a]Inclined unit squares [b]Time in milliseconds.

al.[6] also presented a closed-form solution for the view factor between parallel rectangles, which was later simplified by Ehlert and Smith[7] by eliminating some of the terms in the expression for G that would otherwise get mutually canceled because of the evaluation in Eq. (1). The main advantage of the Gross et al. method[6] is that the coding is much easier compared to the view factor algebra method. The view factor algebra method takes advantage of the special cases where the extensions of the boundary lines meet each other at the $y(\eta)$ axis (when $\eta\text{-}y = 0$), or when a boundary of one or both of the rectangles falls on the $y(\eta)$ axis, since the total number of individual view factors to be obtained is less here.

Numerical performance of the two methods are as follows: 1) the view factor algebra and 2) the Gross et al. method[6] have been carried out. The Simpson rule is a convenient algorithm for numerical integration. However, for an accurate evaluation of the integral, an adaptive quadrature program is used. An adaptive algorithm approximates the integral such that the region where the integrand (function) changes rapidly is divided into finer subintervals for calculation; whereas a coarser grid is used for regions where the function behaves smoothly. This ensures greater efficiency for the algorithm because the number of function evaluations, which is an indication of computational effort, is smaller compared to a nonadaptive scheme, for the same accuracy. For the present calculations, the adaptive integrator Q1DA with automatic error control facility is used. The subroutine Q1DA is based on the Gauss–Kronrod quadrature scheme.[8] Also, calculations were repeated with a 20-panel compound Simpson quadrature formula. All of the numerical experiments were carried out on an IBM PC-AT/386 in single precision arithmetic.

Results and Discussions

The previous methods have been applied to a number of cases and the results have been compared. The results of three cases are presented here: case 1, for the general configuration of offset rectangles (when $\eta\text{-}y \neq 0$); case 2, for the special case (when $\eta\text{-}y = 0$); and case 3, for two squares with one common edge and included angle α for which a solution is already reported in literature.[5]

Case 1: $x_1 = 1, x_2 = 2, y_1 = 1, y_2 = 2, \eta_1 = 2.5, \eta_2 = 3.5, \xi_1 = 2.5, \xi_2 = 3.5$

In this case, 16 view factors are to be evaluated for method 1. The results are given in Table 1 along with the total number of function evaluations (*nfun*) to perform the numerical integration and total CPU times. A simple nonadaptive 20-panel Simpson rule is found to be sufficient for both methods for the purpose of numerical quadrature.

Case 2: $x_1 = 0, x_2 = 1, y_1 = 0, y_2 = 1, \eta_1 = 0, \eta_2 = 1, \xi_1 = 1, \xi_2 = 2$

The results for this case are given in Table 2. For this case, method 1 needs the evaluation of only two view factors for this simple geometry. In this case, method 1 is about six times faster than method 2. Both methods need the use of only a 20-panel Simpson rule for numerical integration.

Case 3: $x_1 = 0, x_2 = 1, y_1 = 0, y_2 = 1, \eta_1 = 0, \eta_2 = 1, \xi_1 = 0, \xi_2 = 1$

This case corresponds to two inclined unit squares having a common edge. For this case, method 1 needs the evaluation of only one view factor. The results for various included angles are given in Table 3. Note that the Gross et al. method[6] tailored with a 20-panel Simpson rule does not give accurate results; whereas the same method when tailored with the adaptive integrator Q1DA gives the same results as those obtained from method 1 tailored with the simple Simpson scheme. The Gross et al. method[6] works at the expense of several function evaluations, and hence, took more CPU time as compared to method 1. This shows that the Gross et al. method[6] requires an accurate numerical integrator that employs schemes with error control, which is not the case for the view factor algebra method, where a very simple quadrature rule worked and produced results of the same accuracy. This is because the integrand in the view factor algebra method is smooth; whereas in the Gross et al. method[6] it is not, for the geometry considered.

Conclusions

For more general configurations of offset rectangles, the view factor algebra method and the Gross et al. method give similar performance in terms of CPU time. However, the view factor algebra method can easily take advantage of simple arrangements, like cases 2 and 3, to reduce the CPU time and it needs the use of only a simple quadrature scheme to produce accurate results. In light of this Note's results, the view factor algebra method is a better choice for calculating the view factor between surfaces for special cases where the lines drawn along the boundaries interest each other, i.e., when $\eta\text{-}y = 0$, or when a boundary of one or both of the rectangles falls on the common $y(\eta)$ axis.

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